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ON A GENERALIZED PROCEDURE FOR THE CALCULATION

OF THERMAL RADIATION TRANSFER

B. T. Chao
Assoc. Prof. of Mech Engrg.
University of Illinois
230 Mech. Engrg. Bldg.
Urbana, Illinois

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ABSTRACT

A systematic procedure is introduced for determining the radiant heat transfer between a radiating gas or flame and a gray receiving surface in a closed chamber consisting of n gray surfaces, all of different emissivities. The effect of radiating beam length on the absorptivity of the gas or flame is considered. The resulting equations are then modified for the calculation of radiant heat transfer between gray surfaces in an enclosure separated by non-absorbing media. The solution and particularly the method presented have wider application than those heretofore published. For numerical calculation, the results can be obtained readily from the digital computer.

ON A GENERALIZED PROCEDURE FOR THE CALCULATION OF THERMAL RADIATION TRANSFER

By B. T. Chao

NOMENCLATURE

The following nomenclature is used in the paper; any consistent system of units can be used for numerical calculation:

A = surface area

$a_{G(ij)}$ = gas absorptivity associated with radiation emitted by A_i and received by A_j

E = total emissive power of a black surface, $E = \sigma T^4$

e_F = equivalent gray-body emissivity of the radiating gas or flame

e_{Gi} = gas emissivity associated with A_i

e_i = normal total emissivity of the gray surface A_i

F_{ij} = shape factor. It is the fraction of the radiant energy emitted or reflected from A_i and received by A_j when there is no gas absorption. In particular, F_{ii} represents the fraction of energy emitted from A_i and intercepted by itself. They are determined completely by the geometry of the system under consideration.

q_{Gi} = gross rate of radiant interchange between the radiating gas or flame and A_i. It is the total amount of thermal radiant energy emitted by the gas and absorbed by A_i in a unit time.

q_{ij} = gross rate of radiant interchange between surfaces A_i and A_j

$q_{G\to i \text{ net}}$ = net rate of radiant interchange between radiating gas or flame and A_i

$q_{i\to j \text{ net}}$ = net rate of radiant interchange between surfaces A_i and A_j

$q_{i,net}$ = net rate of radiant energy leaving A_i

σ = Stephan-Boltzmann radiation constant

T = absolute temperature

${}_iX_r$, ${}_iY_r$, etc. = amount of radiant energy reflected from A_i in a unit time after reflection has taken place for the rth time.

INTRODUCTION

Calculations of radiant heat transfer between a flame and a gray surface, or between two gray surfaces, in the presence of other surfaces which may either be gray or refractory, have been a subject of considerable interest. They arise in the evaluation of combustion-chamber performance, analysis of furnace problems, computation of heat balances of various panel heating and cooling systems, etc. The case which deals with the net radiant interchange between a gray flame and a gray sink due to combined actions of direct radiation, reradiation from the refractory surfaces, thence back through the flame to the sink with partial absorption therein has been treated by Hottel.[1] The combustion chamber considered consists of only one gray sink surrounded by refractory surfaces at a common temperature. Raber and Hutchinson[2] developed a rational expression for the calculation of the thermal energy absorbed at one infinitesimal surface due to emission from another when both could "see" a third. The three surfaces were of different emissivities. Recently, Hottel[3] has published a more general procedure by which the net radiant interchange between two gray surfaces in an enclosure of n gray surfaces can be computed. In the latter two instances, the surfaces were separated by nonabsorbing media. The present analysis forms a generalization of the cases cited and many others which can be found in literature.[3,4,5,6]

DEVELOPMENT OF GENERAL SOLUTION FOR RADIANT INTERCHANGE

BETWEEN A RADIATING GAS OR FLAME AND A GRAY SURFACE

Consider a system consisting of n gray surfaces bounding a radiating gas or flame[7] at a mean temperature T_G. It is desired to calculate the gross and net radiant heat transfer between the gas and any one of the gray surfaces. It will be designated arbitrarily as A_1. The amount of thermal energy emitted by the gas or flame and absorbed at various surfaces in a unit time is listed in Table 1. Kirchhoff's law is assumed valid throughout the present analysis. The reflected energy also has been tabulated and is designated by ${}_1X_1, {}_2X_1, ----{}_nX_1$ at various surfaces as shown. The left-hand subscript refers to the particular surface in question while the right-hand subscript is 1 since they represent thermal energy after the first reflection.

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1 "Heat Transmission," by W. H. McAdams, McGraw-Hill Book Company, Inc., New York, N.Y., 1946, chapter 3.

2 "Panel Heating and Cooling Analysis," by B. F. Raber and F. W. Hutchinson, John Wiley & Sons, Inc., New York, N.Y., 1947, pp. 47-52.

3 "Notes on Radiant Heat Transmission among Surfaces Separated by Non-Absorbing Media," by H. C. Hottel, Department of Chemical Engineering, The Massachusetts Institute of Technology, Cambridge, Mass., revised edition, 46a-d, 1951.

4 O. A. Saunders, Proceedings of the Physical Society of London, vol.41,1929, p.569.

5 E. Eckert, Technische Strahlungsaustauschrechnungen, VDI-Verlag,Berlin,Germany,1937,p.37.

6 "Elements of Heat Transfer and Insulation," by M. Jakob and G. A. Hawkins, John Wiley & Sons, Inc., New York, N.Y., 1950, p. 181.

7 The present analysis will be valid either for a radiating gas or a luminous flame.

The fraction of the reflected energy after once more being absorbed and reflected at various surfaces are given in Table 2. The presence of the terms F_{11}, F_{22}... etc., indicates that the surface may "see" itself. The use of different values of gas absorptivity $A_{G(ij)}$ associated with surfaces A_i and A_j is to account for variation of gas absorptivity with radiating beam length and surface temperature. The summation of all the reflection terms at the surface A_i is now denoted by ${}_iX_2$ where $i = 1, 2, \ldots n$. They will again be absorbed and reflected in precisely the same manner as their predecessors, ${}_iX_1$. Such a multireflection process accompanied by partial absorption due to gas will take place indefinitely in the system under consideration.

From Table 2, it is seen that

$${}_1X_2 = (1 - e_1) \sum_{i=1}^{n} {}_iX_1 F_{i1}(1 - a_{G(i1)})$$

$${}_2X_2 = (1 - e_2) \sum_{i=1}^{n} {}_iX_1 F_{i2}(1 - a_{G(i2)})$$

$$\vdots$$

and

$${}_nX_2 = (1-e_n) \sum_{i=1}^{n} {}_iX_1 F_{in}(1 - a_{G(in)})$$

In general, we can write

$$\left.\begin{aligned}
{}_1X_r &= (1-e_1) \sum_{i=1}^{n} {}_iX_{r-1} F_{i1}(1 - a_{G(i1)}) \\
{}_2X_r &= (1-e_2) \sum_{i=1}^{n} {}_iX_{r-1} F_{i2}(1-a_{G(i2)}) \\
&\vdots \\
\text{and}\quad {}_nX_r &= (1-e_n) \sum_{i=1}^{n} {}_iX_{r-1} F_{in}(1-a_{G(1n)})
\end{aligned}\right\} \quad (1)$$

where $r = 2,3\ldots \infty$.

The thermal energy emitted by the radiating gas and absorbed by A_1 in a unit time due to direct radiation and multireflection and multiabsorption is the sum of all the absorption terms at A_1 as listed in the previous tables. Hence

$$q_{G1} = e_{G1}E_G e_1 A_1 + e_1\Big\{F_{11}(1-a_{G(11)}) \sum_{s=1}^{\infty} {}_1X_s + F_{21}(1 - a_{G(21)}) \sum_{s=1}^{\infty} {}_2X_s$$

$$+ \dots + F_{n1}(1-a_{G(n1)}) \sum_{s=1}^{\infty} {}_nX_s\Big\} \quad . \quad . \quad . \quad . \qquad (2)$$

We shall now proceed to evaluate the unknowns $\sum_{s=1}^{\infty} {}_iX_s$, with $i=1, 2 \dots n$. From Equations (1), the following set of equations can be readily deduced:

$$\left.\begin{aligned}
\sum_{s=1}^{\infty} {}_1X_s &= {}_1X_1 + (1-e_1) \sum_{i=1}^{n} F_{i1}\ (1-a_{G(i1)}) \sum_{s=1}^{\infty} {}_iX_s \\
\sum_{s=1}^{\infty} {}_2X_s &= {}_2X_1 + (1-e_2) \sum_{i=1}^{n} F_{i2}\ (1-a_{G(i2)}) \sum_{s=1}^{\infty} {}_iX_s \\
&\vdots \\
\sum_{s=1} {}_nX_s &= {}_nX_1 + (1-e_n) \sum_{i=1}^{n} F_{in}\ (1-a_{G(in)}) \sum_{s=1}^{\infty} {}_iX_s
\end{aligned}\right\} (3)$$

For simplicity, we designate

$${}_1\underline{X} = \sum_{s=1}^{\infty} {}_1X_s, \quad {}_2\underline{X} = \sum_{s=1}^{\infty} {}_2X_s$$

and

$${}_n\underline{X} = \sum_{s=1}^{\infty} {}_nX_s$$

Also, from Table I, it is seen that

$${}_1X_1 = e_{G1}\ E_G\ A_1(1-e_1), \qquad {}_2X_1 = e_{G2}\ E_G\ A_2(1-e_2), \text{ etc.}$$

Substituting these relationships into the set of Equations (3) and rearranging, gives

$$
\left.
\begin{aligned}
&{}_1\underline{X}\left\{F_{11}[1-a_{G(11)})-\frac{1}{1-e_1}\right\}+{}_2\underline{X}\,F_{21}(1-a_{G(21)}]+{}_3\underline{X}\,F_{31}[1-a_{G(31)}]\\
&\qquad+\dots+{}_n\underline{X}\,F_{n1}[1-a_{G(n1)}]=-e_{G1}E_GA_1\\
&{}_1\underline{X}\,F_{12}[1-a_{G(12)}]+{}_2\underline{X}\left\{F_{22}[1-a_{G(22)}]-\frac{1}{1-e_2}\right\}+{}_3\underline{X}\,F_{32}[1-a_{G(32)}]\\
&\qquad+\dots+{}_n\underline{X}\,F_{n2}[1-a_{G(n2)}]=-e_{G2}E_GA_2\\
&\qquad\cdot\\
&{}_1\underline{X}\,F_{13}[1-a_{G(13)}]+{}_2\underline{X}\,F_{23}[1-a_{G(23)}]+{}_3\underline{X}\left\{F_{33}[1-a_{G(33)}]-\frac{1}{1-e_3}\right\}\\
&\qquad+\dots+{}_n\underline{X}\,F_{n3}[1-a_{G(n3)}]=-e_{G3}E_GA_3\\
&{}_1\underline{X}\,F_{1n}[1-a_{G(1n)}]+{}_2\underline{X}\,F_{2n}[1-a_{G(2n)}]+{}_3\underline{X}\,F_{3n}[1-a_{G(3n)}]\\
&\qquad+\dots+{}_n\underline{X}\left\{F_{nn}[1-a_{G(nn)}]-\frac{1}{1-e_n}\right\}=-e_{Gn}E_GA_n
\end{aligned}
\right\}\quad(4)
$$

We have thus n linear algebraic equations to be solved simultaneously for n unknowns: namely, ${}_1\underline{X}$, ${}_2\underline{X}$, ... and ${}_n\underline{X}$. Standard programs are available for the solution of these equations on the digital computer.[8]

$$
q_{G1}=e_{G1}E_Ge_1A_1+e_1\sum_{i=1}^{n}F_{i1}\,(1-a_{G(i1)}\;{}_i\underline{X}\qquad\ldots\ (5)
$$

8 The "Illiac," which is the digital computing machine at the University of Illinois can handle such problems up to n=39.

In combustion-chamber analysis, it is often desirable to find the "net" radiant interchange between the radiating gas or flame and a cold receiving surface. To this end, we shall now formulate an expression for q_{1G}, the thermal energy emitted by A_1 and absorbed by the gas due to direct and reflected radiation.

A procedure analogous to that employed for the calculation of q_{G1} will again be followed. Table 3 gives the radiant energy absorbed by the gas and the various surface reflections after once being received from the radiation emitted from A_1. The reflection terms are designated by ${}_iy_1$, with i varying from 1 to n. Similarly, Table 4 illustrates the gas absorption and the energy reflected at various surfaces after once more being received from the reflected energy at all surfaces listed in Table 3. As before, such a multiabsorption and reflection process will take place indefinitely in the closed chamber.

From the preceding tabulation, the following relationships can be readily deduced:

$$\left.\begin{aligned}
{}_1Y_r &= (1-e_1) \sum_{i=1}^{n} {}_iY_{r-1} F_{i1} \left[1 - a_{G(i1)}\right] \\
{}_2Y_r &= (1-e_2) \sum_{i=1}^{n} {}_iY_{r-1} F_{i2} \left[1 - a_{G(i2)}\right] \\
&\vdots \\
{}_nY_r &= (1-e_n) \sum_{i=1}^{n} {}_iY_{r-1} F_{in} \left[1 - a_{G(in)}\right]
\end{aligned}\right\} \quad (6)$$

where $r = 2, 3 \ldots\ldots \infty$.

The thermal energy emitted by A_1 and absorbed by the gas in a unit time is the sum of all the terms under the gas-absorption columns, the first two of which are illustrated in Tables 3 and 4, respectively. Hence

$$q_{1G} = e_1 E_1 A_1 \left\{ \left(1 + \sum_{s=1}^{\infty} {}_1Y_s\right) \sum_{i=1}^{n} F_{1i} a_{G(1i)} + \left(\sum_{s=1}^{\infty} {}_2Y_s\right) \sum_{i=1}^{n} F_{2i} a_{G(2i)} + \ldots + \left(\sum_{s=1}^{\infty} {}_nY_s\right) \sum_{i=1}^{n} F_{ni}\, a_{G(ni)} \right\} \quad . \quad . \quad . \quad . \quad (7)$$

Following a procedure similar to that used for evaluating $\sum_{s=1}^{\infty} {}_iX_s$; and also, by writing ${}_iY$ for $\sum_{s=1}^{\infty} {}_iY_s$, with i varying from 1 to n, one can deduce the following set of equations:

$$\left.\begin{aligned}
&{}_1\underline{Y}\left\{F_{11}\left[1 - a_{G(11)}\right] - \frac{1}{1-e_1}\right\} + {}_2\underline{Y}\,F_{21}\left[1-a_{G(21)}\right] + {}_3\underline{Y}\,F_{31}\left[1-a_{G(31)}\right] \\
&\qquad + \ldots + {}_n\underline{Y}\,F_{n1}\left[1-a_{G(n1)}\right] = -F_{11}\left[1-a_{G(11)}\right] \\
&{}_1\underline{Y}\,F_{12}\left[1-a_{G(12)}\right] + {}_2\underline{Y}\left\{F_{22}\left[1-a_{G(22)}\right] - \frac{1}{1-e_2}\right\} + {}_3\underline{Y}\,F_{32}\left[1-a_{G(32)}\right] \\
&\qquad + \ldots + {}_n\underline{Y}\,F_{n2}\left[1-a_{G(n2)}\right] = -F_{12}\left[1-a_{G(12)}\right] \\
&{}_1\underline{Y}\,F_{13}\left[1-a_{G(13)}\right] + {}_2\underline{Y}\,F_{23}\left[1-a_{G(23)}\right] + {}_3\underline{Y}\left\{F_{33}\left[1-a_{G(33)}\right] - \frac{1}{1-e_3}\right\} \\
&\qquad + \ldots + {}_n\underline{Y}\,F_{n3}\left[1-a_{G(n3)}\right] = -F_{13}\left[1-a_{G(13)}\right] \\
&{}_1\underline{Y}\,F_{1n}\left[1-a_{G(1n)}\right] + {}_2\underline{Y}\,F_{2n}\left[1-a_{G(2n)}\right] + {}_3\underline{Y}\,F_{3n}\left[1-a_{G(3n)}\right] \\
&\qquad + \ldots + {}_n\underline{Y}\left\{F_{nn}\left[1-a_{G(nn)}\right] - \frac{1}{1-e_n}\right\} = -F_{1n}\left[1-a_{G(1n)}\right]
\end{aligned}\right\} \quad (8)$$

Again, we have n linear algebraic equations to be solved simultaneously. It may be remarked that the coefficients of the unknown ${}_1\underline{Y}$, ${}_2\underline{Y}$, etc. are identical to the corresponding ones in Equations (4). With these unknowns solved, Equation (7) may be modified as follows:

$$\left.\begin{aligned}
q_{1G} = e_1E_1A_1 &\left\{(1 + {}_1\underline{Y}) \sum_{i=1}^{n} F_{1i}\, a_{G(1i)} + {}_2\underline{Y} \sum_{i=1}^{n} F_{2i}\, a_{G(2i)}\right. \\
&\left. + \ldots + {}_n\underline{Y} \sum_{i=1}^{n} F_{ni}\, a_{G(ni)}\right\}
\end{aligned}\right\} \quad (9)$$

If it can be assumed that the gas or flame is gray, i.e., an equivalent gray-body emissivity e_F can be adopted which serves both as absorptivity and emissivity, then

$$a_{G(11)} = A_{G(12)} = a_{G(11)} \cdots\cdots \text{ etc.} \quad = e_F$$

And, since $\sum_{i=1}^{n} F_{1i} = 1$, $\sum_{i=1}^{n} F_{2i} = 1$, etc., Equation (9) simplifies to

$$q_{1G} = e_1 E_1 A_1 e_F \left(1 + \sum_{i=1}^{n} {}_i\underline{Y}\right) \quad . \quad . \quad . \quad . \quad . \quad (9a)$$

Consequently, the net radiant interchange between the radiating gas and the gray surface A_1 is

$$\underset{\text{net}}{q_{G\rightarrow 1}} = q_{G1} - q_{1G} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

<u>Illustration</u>

Hottel[3] solved a simple case in which the combustion chamber consists only of two surfaces. They are the cold receiving surface A_c, at a uniform temperature T_c, and the refractory wall A_R also assumed to have a uniform temperature.

The flame is gray and a mean temperature T_F is assignable to it. To apply Equations (5) and (9a), we set $n = 2$, $A_1 = A_c$, $A_2 = A_R$, $e_1 = e_c$, $e_2 = e_R = 0$, $F_{11} = F_{cc}$, $F_{12} = F_{cR}$, $F_{21} = F_{Rc}$, $F_{22} = F_{RR}$ and replaces all the a_G by e_F, the equivalent emissivity (= absorptivity) of the flame. Hence

$$q_{Fc} = e_F E_F e_c A_c + e_c\ (1 - e_F)(F_{cc}\ {}_1\underline{X} + F_{Rc}\ {}_2\underline{X}) \quad . \quad . \quad . \quad (11)$$

where

$${}_1\underline{X} = - e_F E_F \frac{\begin{vmatrix} A_c & F_{Rc} & (1-e_F) \\ A_R & F_{RR} & (1-e_F)-1 \end{vmatrix}}{\Delta} \quad . \quad . \quad . \quad (11a)$$

$$ {}_2X = -e_F E_F \frac{\begin{vmatrix} F_{cc}(1-e_F) - \frac{1}{1-e_c} & A_c \\ F_{cR}(1-e_F) & A_R \end{vmatrix}}{\Delta} \quad \ldots\ldots \quad (11b) $$

and

$$ \Delta = \begin{vmatrix} F_{cc}(1-e_f) - \frac{1}{1-e_c} & F_{Rc}(1-e_F) \\ F_{cR}(1-e_F) & F_{RR}(1-e_F)-1 \end{vmatrix} \quad \ldots \quad (11c) $$

Similarly, by Equation (9a) we have

$$ q_{cF} = e_c E_c A_c e_F(1 + {}_1Y + {}_2Y) \quad \ldots\ldots \quad (12) $$

where

$$ {}_1Y = -(1-e_F) \frac{\begin{vmatrix} F_{cc} & F_{Rc}(1-e_F) \\ F_{cR} & F_{RR}(1-e_F) - 1 \end{vmatrix}}{\Delta} \quad \ldots\ldots \quad (12a) $$

$$ {}_2Y = -(1-e_F) \frac{\begin{vmatrix} F_{cc}(1-e_F) - \frac{1}{1-e_c} & F_{cc} \\ F_{cR}(1-e_F) & F_{cR} \end{vmatrix}}{\Delta} \quad \ldots\ldots \quad (12b) $$

and Δ is given by Equation (11c).

Substituting Equations (11) and (12) into (10) and simplifying yields the following expression for the "net" radiant interchange between the flame and the cold receiving surface due to combined mechanisms of direct radiation from the flame, reradiation from the refractory wall, multireflection and absorption inside the chamber.

$$q_{\underset{net}{F \to c}} = q_{Fc} - q_{cF} = (E_F - E_c) A_c \mathcal{F}_{cF}$$

$$= \sigma (T_F^4 - T_c^4) A_c \mathcal{F}_{cF} \quad . \quad . \quad . \quad (13)$$

where

$$\mathcal{F}_{cF} = \frac{1}{\frac{1}{\bar{e}_{cF}} + \frac{1}{e_c} - 1}$$

and

$$\bar{e}_{cF} = e_F \left(1 + \frac{A_R/A_C}{1 + \frac{e_F}{1-e_F} \cdot \frac{1}{F_{Rc}}} \right)$$

which is identical to Hottel's result.

APPLICATION OF GENERAL SOLUTION TO AN ENCLOSURE IN WHICH SURFACES ARE SEPARATED BY NONABSORBING MEDIA

In this case, a problem of usual interest is the calculation of both the gross and net radiant heat transfer between any two of them. These surfaces shall arbitrarily be designated as A_1 and A_2. Again, an enclosure of n gray surfaces all of different emissivities and at different temperatures will be considered.

From the foregoing analysis, it is evident that the reflected radiant energy at various surfaces after once being received from the emission due to A_1 is identical to those given in Table 3, provided that all the a_G are set to zero. Similarly, the reflected energy after once more being reflected at various surfaces is again given by Table 4, on setting $a_G = 0$. Consequently, if ${}_i\underline{Z}$ is written for $\sum_{s=1}^{\infty} {}_iZ_s$ (i = 1,2 ...n) where ${}_iZ_r$ represents the amount of radiant energy reflected from a surface A_i in a unit time after reflection has taken place for the rth time, a set of n linear algebraic equations in ${}_i\underline{Z}$ is readily obtained by a procedure identical to that for ${}_i\underline{Y}$. In fact, they are identical to Equations (8) if one writes, ${}_i\underline{Z}$ for ${}_i\underline{Y}$ and at the same time, sets all the a_G to zero. Thence, it is easy to deduce

$$q_{12} = e_1 E_1 A_1 e_2 \left(F_{12} + \sum_{i=1}^{n} {}_i\underline{Z}\, F_{i2}\right) \quad . \quad . \quad . \quad . \quad . \quad (14)$$

which is the gross rate of radiant heat transfer from A_1 to A_2. The net radiant interchange between A_1 and A_2 is

$$q_{\underset{net}{1\to 2}} = q_{12} - q_{21} = (E_1 - E_2)A_1e_1e_2\left(F_{12} + \sum_{i=1}^{n} {}_{1}\underline{Z}\, F_{i2}\right) \quad . \quad . \quad (15)$$

Many examples which illustrate the application of (14) and (15) can be found elsewhere.[9]

Obviously, the net radiant flux leaving A_1 in an enclosure of 'n' gray surfaces can be computed from either of the following expressions:

$$q_{1,net} = A_1E_1e_1 - \sum_{i=1}^{n} q_{i1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16a)$$

or

$$q_{1,net} = q_{\underset{net}{1\to 2}} + q_{\underset{net}{1\to 3}} + \cdots + q_{\underset{net}{1\to n}} \quad . \quad . \quad . \quad . \quad (16b)$$

DISCUSSION

In the formulation of the expression for q_{G1}, q_{1G}, q_{12} etc., it has been tacitly assumed that the integral of the infinitesimal shape factor products can be replaced by the finite shape factor products. Had infinitesimal areas and their corresponding shape factors been used, the foregoing analysis would be regorously true under the assumptions stated. The inequality between shape factors for finite and for infinitesimal areas has been mentioned by Hottel and quantitatively investigated by Raber and Hutchinson.[2] However, the present analysis enables one to obtain results with an accuracy as high as one desires, since the solution is valid for n surfaces and n can be chosen as large as desired. The larger the number of subdivisions of the area, the smaller the error will be.

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[9] "Generalized Interchange Factor for Calculating Radiant Heat Transfer," by E. L. Sartain, Master of Science thesis, Department of Mechanical Engineering, University of Illinois, Urbana, Ill., January, 1954.

Table 1 Absorbed and Reflected Radiant Thermal Energy at Various Surfaces after Once Being Received from the Radiating Gas or Flame

A_1		A_2		- - -	A_n	
Absorption	Reflection	Absorption	Reflection		Absorption	Reflection
$e_{G1}E_GA_1e_1$	$e_{G1}E_GA_1(1-e_1)$ $= {}_1X_1$	$e_{G2}E_GA_2e_2$	$e_{G2}E_GA_2(1-e_2)$ $= {}_2X_1$		$e_{Gn}E_GA_ne_n$	$e_{Gn}E_GA_n(1-e_n)$ $= {}_nX_1$

Table 2 Absorbed and Reflected Radiant Thermal Energy at Various Surfaces after Once Being Received from the Reflected Energy at All Surfaces Listed in Table 1, with Partial Absorption Due to Gas, Therein

A_1		A_2		- - -	A_n	
Absorption	Reflection	Absorption	Reflection		Absorption	Reflection
${}_1X_1F_{11}(1-a_{G(11)})e_1$	${}_1X_1F_{11}(1-a_{G(11)})(1-e_1)$	${}_1X_1F_{12}(1-a_{G(12)})e_2$	${}_1X_1F_{12}(1-a_{G(12)})(1-e_2)$		${}_1X_1F_{1n}(1-a_{G(1n)})e_n$	${}_1X_1F_{1n}(1-a_{G(1n)})(1-e_n)$
${}_2X_1F_{21}(1-a_{G(21)})e_1$	${}_2X_1F_{21}(1-a_{G(12)})(1-e_1)$	${}_2X_1F_{22}(1-a_{G(22)})e_2$	${}_2X_1F_{22}(1-a_{G(22)})(1-e_2)$		${}_2X_1F_{2n}(1-a_{G(2n)})e_n$	${}_2X_1F_{2n}(1-a_{G(2n)})(1-e_n)$
.	.	.	.		.	.
.	.	.	.		.	.
.	.	.	.		.	.
${}_nX_1F_{n1}(1-a_{G(n1)})e_1$	${}_nX_1F_{n1}(1-a_{G(n1)})(1-e_1)$	${}_nX_1F_{n2}(1-a_{G(n2)})e_2$	${}_nX_1F_{n2}(1-a_{G(n2)})(1-e_2)$		${}_nX_1F_{nn}(1-a_{G(nn)})e_n$	${}_nX_1F_{nn}(1-a_{G(nn)})(1-e_n)$
	$\Sigma = {}_1X_2$		$\Sigma = {}_2X_2$			$\Sigma = {}_nX_2$

Table 3 Gas Absorption and Reflected Radiant Energy at Various Surfaces after Once Being Received from the Emission Due to A_1

Gas Absorption	Surface Reflection		- - -	
	A_1	A_2		A_n
$F_{11}a_{G(11)}$				
$F_{12}a_{G(12)}$				
.	$F_{11}(1-a_{G(11)})(1-e_1)$	$F_{12}(1-a_{G(12)})(1-e_2)$		$F_{1n}(1-a_{G(1n)})(1-e_n)$
.	$= {}_1Y_1$	$= {}_2Y_1$		$= {}_nY_1$
.				
$F_{1n}a_{G(1n)}$				
$\sum_{i=1}^{n} F_{1i}a_{G(1i)}$				

The radiant thermal energy emitted by A_1 in a unit time ($= A_1E_1e_1$) is taken as unity.

Table 4 Gas Absorption and Reflection at Various Surfaces after Once Being Received from the Reflected Energy at All Surfaces Given in Table 3

Gas Absorption	Surface Reflection			
	A_1	A_2	--	A_n
${}_1Y_1F_{11}a_{G(11)}$ ${}_1Y_1F_{12}a_{G(12)}$ . . . ${}_1Y_1F_{1n}a_{G(1n)}$	${}_1Y_1F_{11}(1-a_{G(11)})(1-e_1)$	${}_1Y_1F_{12}(1-a_{G(12)})(1-e_2)$		${}_1Y_1F_{1n}(1-a_{G(1n)})(1-e_n)$
${}_2Y_1F_{21}a_{G(21)}$ ${}_2Y_1F_{22}a_{G(22)}$ . . . ${}_2Y_1F_{2n}a_{G(2n)}$	${}_2Y_1F_{21}(1-a_{G(21)})(1-e_1)$	${}_2Y_1F_{22}(1-a_{G(22)})(1-e_2)$		${}_2Y_1F_{2n}(1-a_{G(2n)})(1-e_n)$
etc.				
${}_nY_1F_{n1}a_{G(n1)}$ ${}_nY_1F_{n2}a_{G(n2)}$ . . . ${}_nY_1F_{nn}a_{G(nn)}$	${}_nY_1F_{n1}(1-a_{G(n1)})(1-e_1)$	${}_nY_1F_{n2}(1-a_{G(n2)})(1-e_2)$		${}_nY_1F_{nn}(1-a_{G(nn)})(1-e_n)$
	$\Sigma = {}_1Y_2$	$\Sigma = {}_2Y_2$		$\Sigma = {}_nY_2$